Kognitionspsychologie: Session 5

Numerical Cognition

Rui Mata, HS2025

Version: November 4, 2025

Learning Objectives

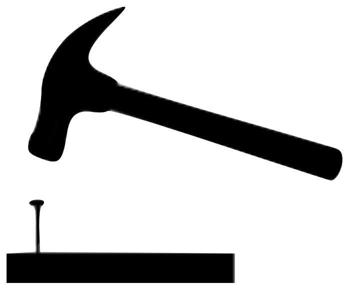
- Be able to discuss whether numerical cognition is a natural kind…
- Discuss the **adaptive significance** of numerical cognition in the past and today (using gender differences as an example of ist importance)
- Learn how **comparative approaches** help understand core components of numerical cognition (e.g., approximate number system)
- Learn about **developmental patterns** concerning numerical abilities, including those associated the representation of small magnitudes (e.g., approximate number system) up to more complex arithmetic concepts and operations
- Learn about **neural model**(s) of numerical cognition

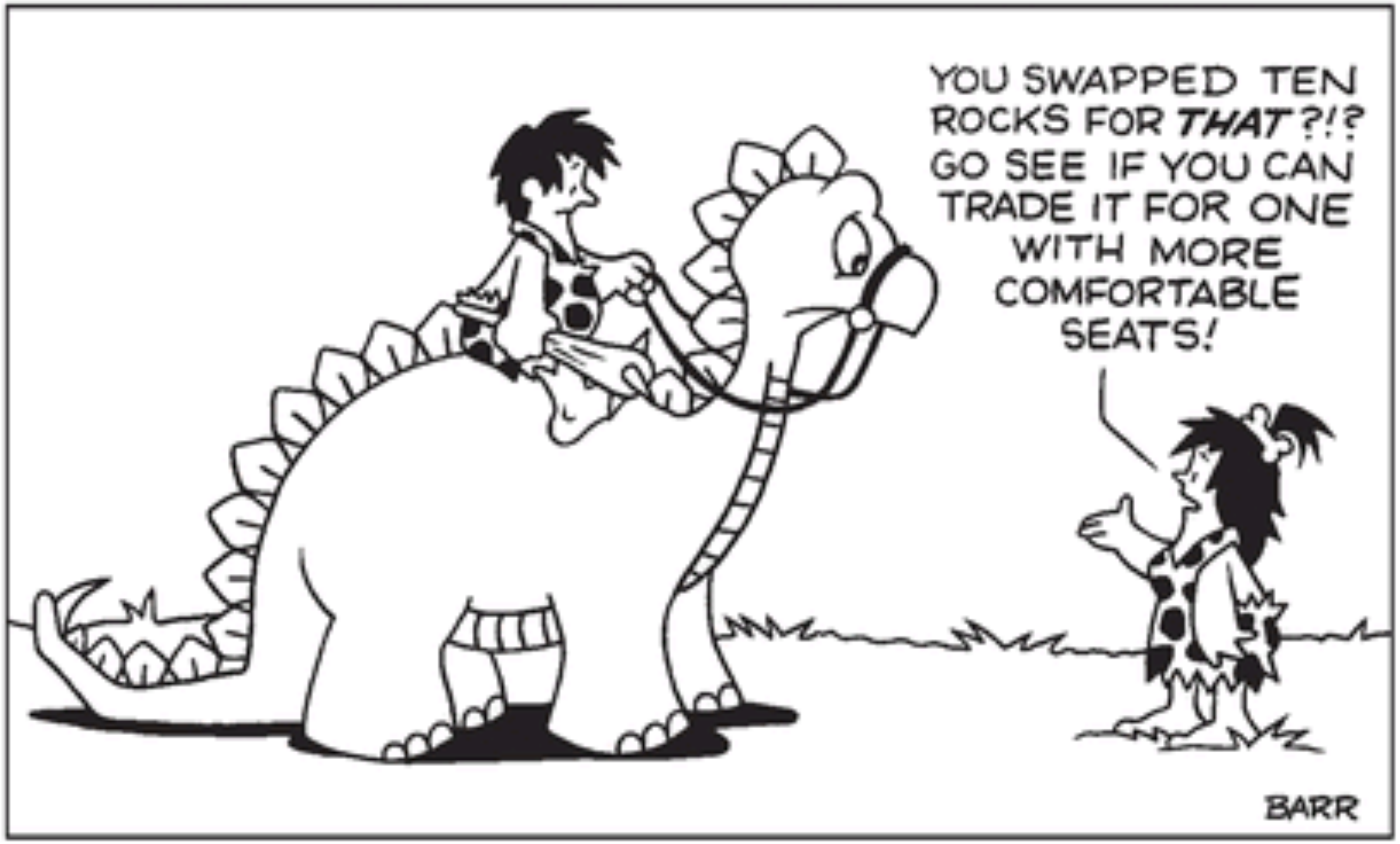
Do you see more blue or yellow beads?

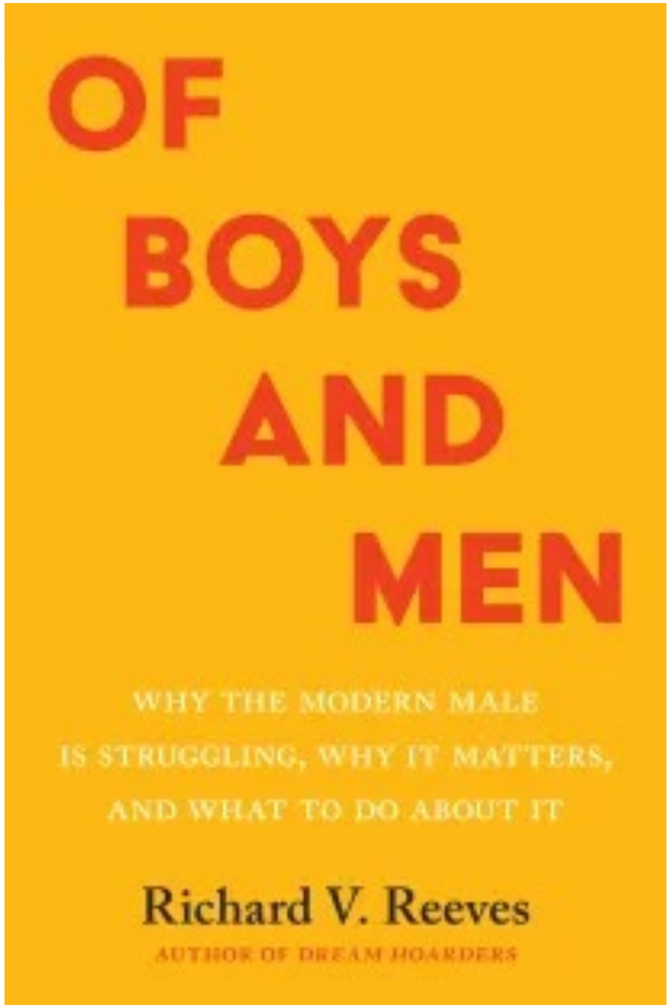
How much is 87 + 34?

What is compound interest?

Numerical Cognition

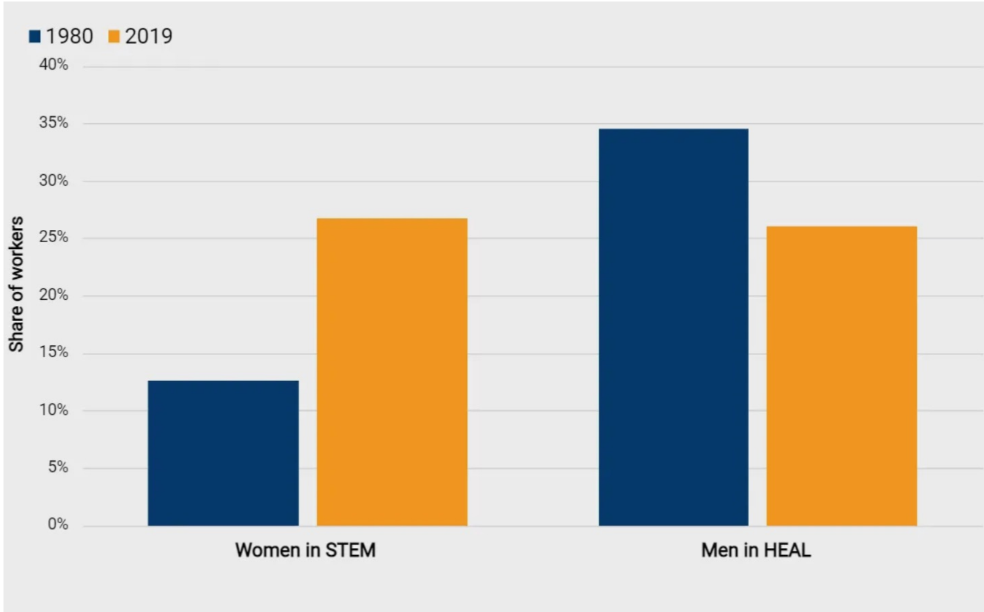
Ontogeny	Mechanism
Phylogeny	 Adaptive Significance


YOU SWAPPED TEN ROCKS FOR *THAT*?!? GO SEE IF YOU CAN TRADE IT FOR ONE WITH MORE COMFORTABLE SEATS!
BARR



STEM v. HEAL

(science, technology, engineering, mathematics) (health, education, administration, literacy)



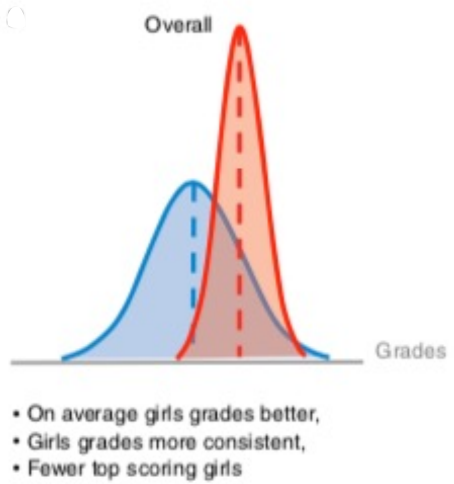
Note that these figures are for full-time, prime age (25-54) workers, CPS data

The debate concerning the causes for gender differences in numerical cognition is as topical as ever…

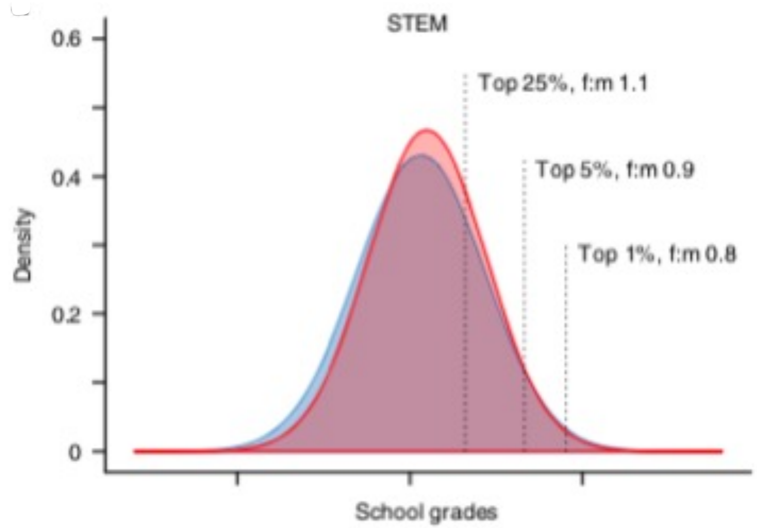
Gender differences in STEM

Fewer women than men pursue careers in science, technology, engineering and mathematics (STEM), despite many girls outperforming boys at school in the relevant subjects.

Variability Hypothesis



Empirical distribution

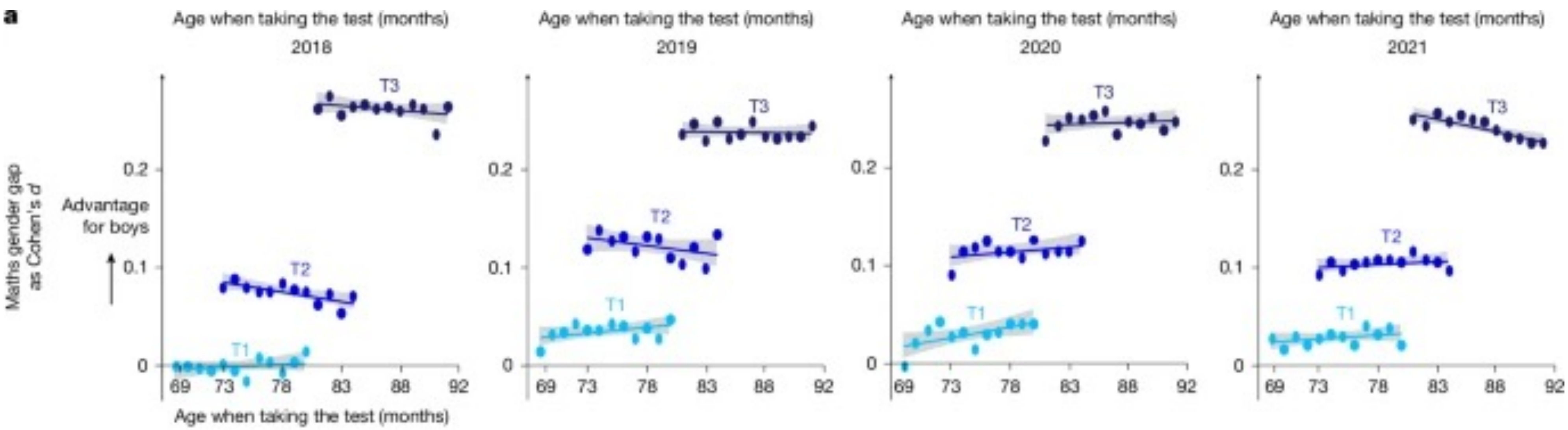


O'Dea et al. find (meta-analytic) evidence for lower variation among girls than boys, and higher average grades for girls: "While our results support the variability hypotheses, we have shown that the magnitude of the gender gap in STEM grades is small, and only becomes male-skewed at the very top of the distribution. Therefore, by the time a girl graduates, she is just as likely as a boy to have earned high enough grades to pursue a career in STEM. When she evaluates her options, however, the STEM path is trod by more male competitors than non-STEM, and presents additional internal and external threats due to her and societies' gendered beliefs (stereotype threat and backlash effects)."

O'Dea, R. E., Lagisz, M., Jennions, M. D., & Nakagawa, S. (2018). Gender differences in individual variation in academic grades fail to fit expected patterns for STEM. Nature Communications, 9(1), 3777. http://doi.org/10.1038/s41467-018-06292-0

Gender differences in STEM

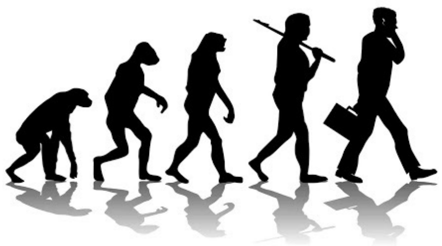
Some studies suggest gender gaps favoring boys emerging with early schooling (rather than age).



Martinot et al. examined over 2.6 million children in France: “Boys and girls exhibited very similar maths scores upon school entry, but a gender gap in favour of boys became highly significant after 4 months of schooling and reached an effect size of about 0.20 after 1 year. These findings were repeated each year and varied only slightly across family, class or school type and socio-economic level. Although schooling correlated with age, exploiting the near-orthogonal variations indicated that the gender gap increased with schooling rather than with age. These findings point to the first year of school as the time and place where a maths gender gap emerges in favour of boys, thus helping focus the search for solutions and interventions.”

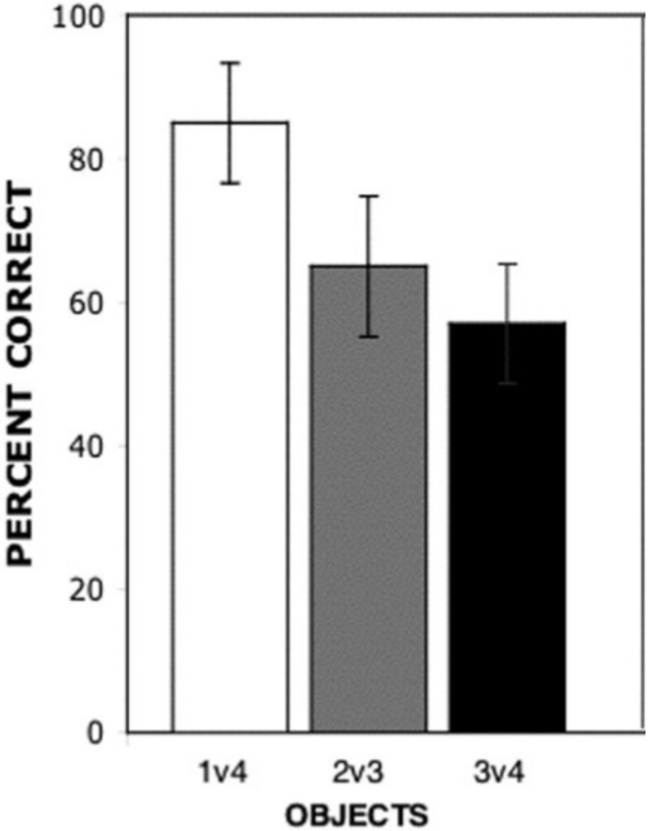
Martinot, P., Colnet, B., Breda, T. et al. Rapid emergence of a maths gender gap in first grade. Nature 643, 1020–1029 (2025). https://doi.org/10.1038/s41586-025-09126-4

Numerical Cognition

Ontogeny	Mechanism
 Phylogeny	Adaptive Significance

Number: Comparative Approach

Capuchin monkey



Numerosity

The approximate sense of numerical quantities, such as the idea that a collection of 4 objects represents a larger quantity than 3, 2, or 1.

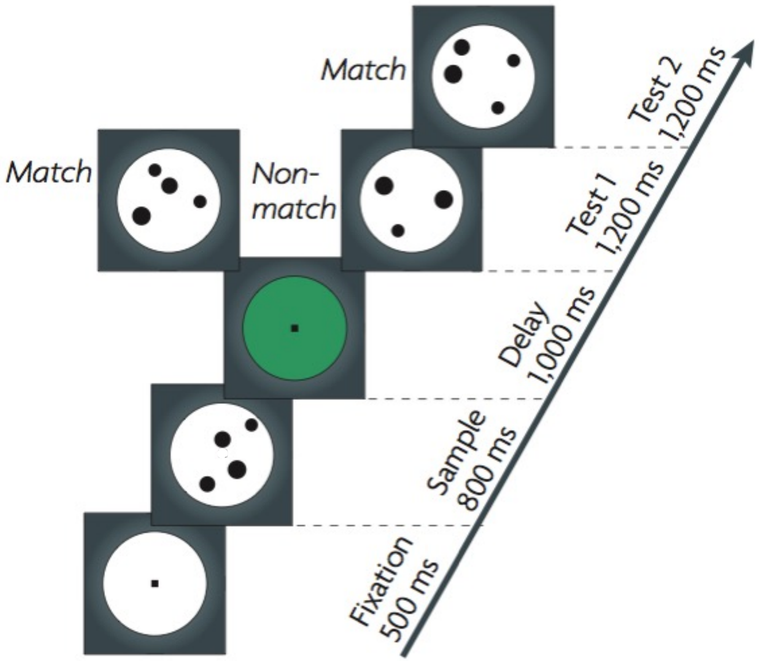
A variety of studies have demonstrated that non-human animals, including various mammals and non-human primates have an approximate sense of number (i.e., numerosity). In this example, capuchins monkey are shown to reliably (but not perfectly) discriminate between small quantities.

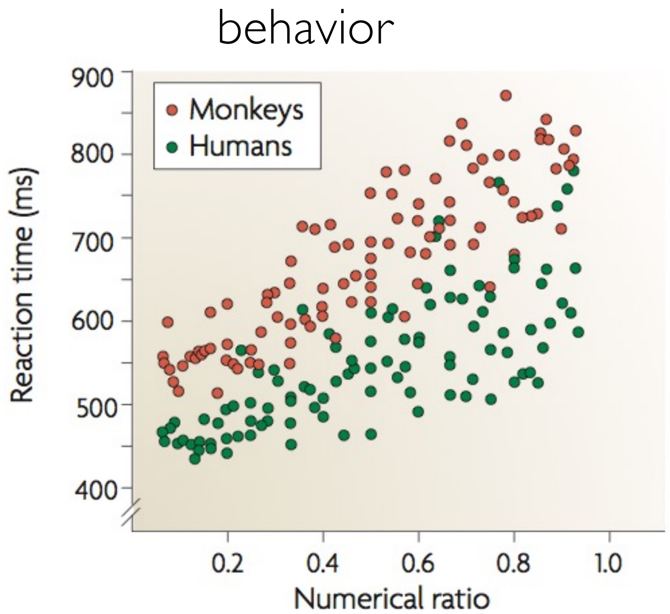
vanMarle, K., Aw, J., McCrink, K. & Santos, L. (2006). How Capuchin monkeys (Cebus apella) quantify objects and substances. ***Journal of Comparative Psychology, 120***, 416-426.

Number: Comparative Approach

Delayed Match-to-Sample Task

Participants are asked to first encode a stimulus (e.g., number of points), and later make a forced-choice response among options where one corresponds to that stimulus.





Comparative studies reveal that both humans and non-human primates are sensitive to different magnitudes can represent them and show similar behavioural phenomena (e.g., ratio effect).

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews Neuroscience, 9*, 278–291.

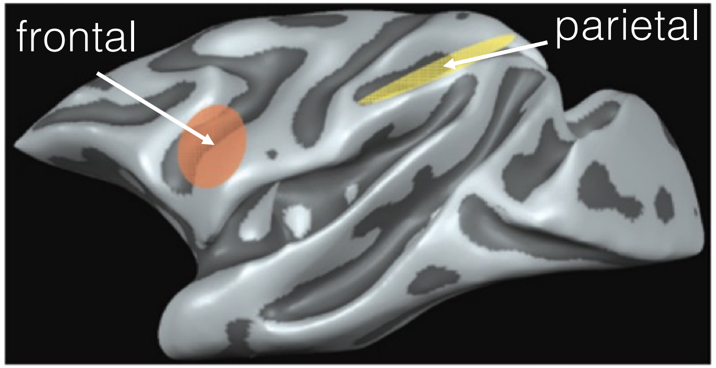
Number: Comparative Approach

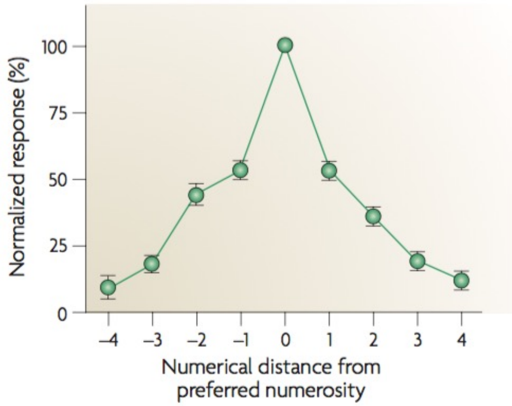
Delayed Match-to-Sample Task

Participants are asked to first encode a stimulus (e.g., number of points), and later make a forced-choice response among options where one corresponds to that stimulus.

Rhesus monkey

neural activity





Studies have revealed the existence of 'number neurons' in the parietal cortex which fire preferentially to the presentation of a particular number of dots. Further studies suggest that both frontal and parietal regions are involved in processing magnitudes.

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews Neuroscience, 9*, 278–291.

Numerical Cognition

Ontogeny 	Mechanism
Phylogeny	Adaptive Significance

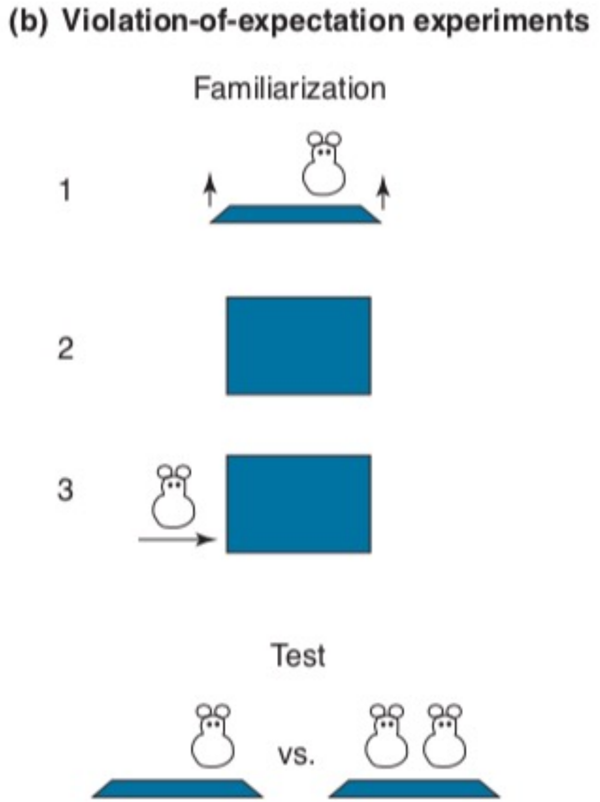
The Development of Arithmetic Abilities

The figure exemplifies discrimination of the numerosities 8 vs. 16 using a habituation paradigm. Infants first see repeated presentations of either 8 or 16 dots. When tested with alternating arrays of 8 and 16 dots, infants (6-months and older) typically look longer at the numerically novel test arrays regardless of whether they had been habituated to 8 or 16, showing that they successfully responded to number. However, studies suggest that infants have problems distinguishing between smaller ratios (e.g., 8 vs. 12).

Feigenson et al. conclude that such results suggest that a core system is already online in infants for representing approximate numerical magnitudes - **approximate number system**.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. ***Trends in Cognitive Sciences, 8***(7), 307–314. http://doi.org/10.1016/j.tics.2004.05.002

The Development of Arithmetic Abilities



Violation-of-expectation experiments can be used to test infants expectations about simple arithmetic. Infants as young as 6 or 9-months old recognise ordinal relationships between numerosities and form expectations about the outcomes of simple arithmetic problems such as as the one presented in the figure (1+1).

Feigenson et al. suggest that a core system is online in infants for representing such operations based on precise representations of distinct individuals (up to 3) - **object identification system**.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. ***Trends in Cognitive Sciences, 8***(7), 307–314. http://doi.org/10.1016/j.tics.2004.05.002

The Development of Arithmetic Abilities

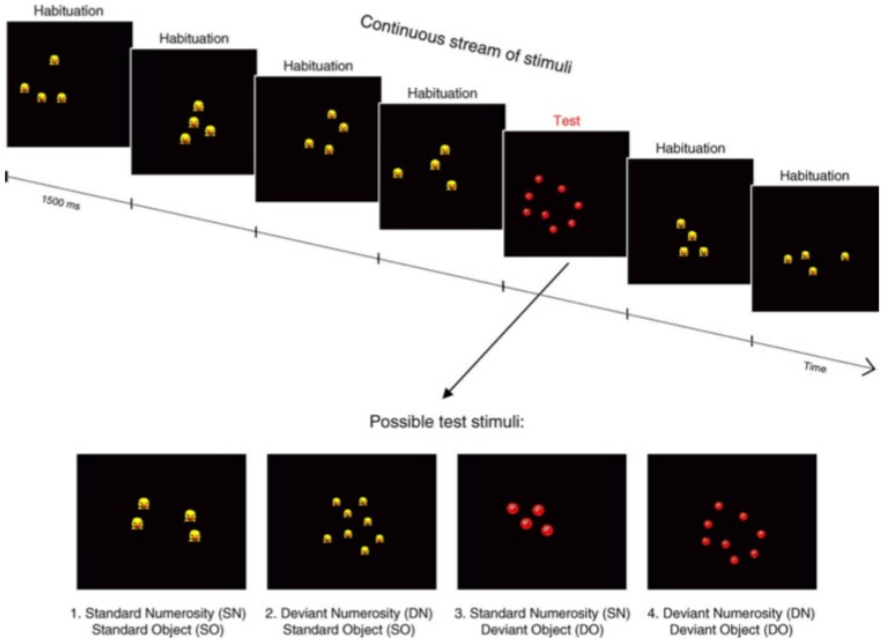
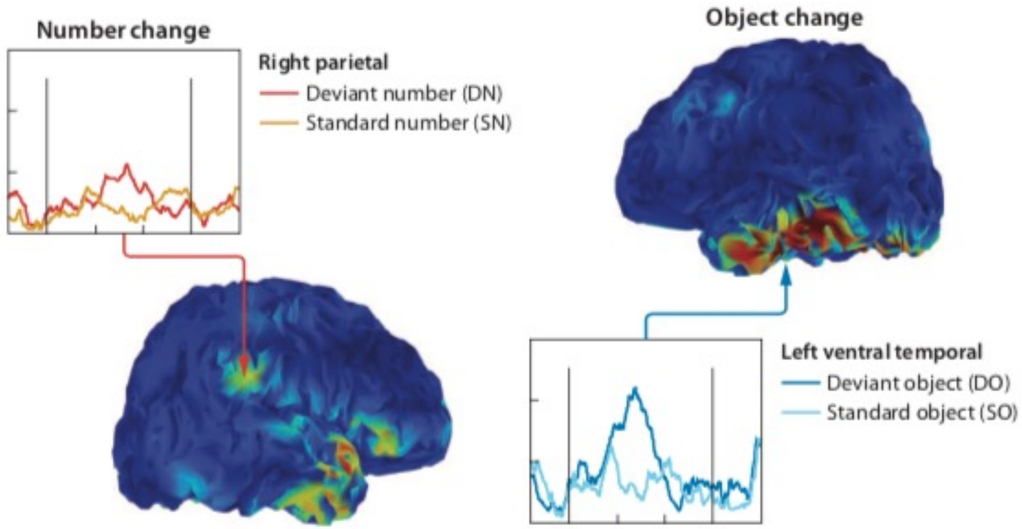


Figure 1. Schematic Description of the Experimental Protocol
Infants were presented with a continuous stream of images, each depicting a set of objects. Within a given block, all habituation stimuli shared the same SN and object identity. Occasionally, a test stimulus was inserted that could differ in number and/or object identity. We only report ERPs to those test stimuli, averaged separately for standard versus deviant number trials, and for standard versus deviant object trials.
doi:10.1371/journal.pbio.0060011.g001

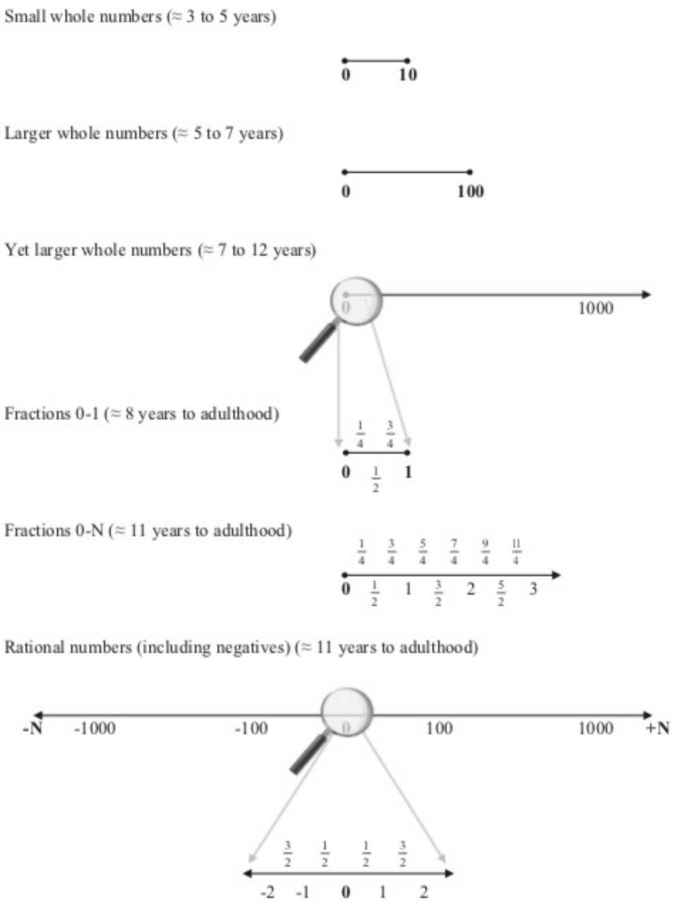


Neuroimaging experiments (e.g., Izard et al., 2008) have used event-related potentials to monitor infant's brain responses to changes in numerosity or object identity. In an experiment with 3-month-old infants, right parietal cortex responded strongly to number change whereas left ventral temporal cortex responded strongly to changes in object identity.

Nieder, A., & Dehaene, S. (2009). Representation of Number in the Brain. ***Annual Review of Neuroscience***, ***32***(1), 185–208. http://doi.org/10.1146/annurev.neuro.051508.135550

The Development of Arithmetic Abilities

Magnitude representations (number line)



Strategies/Operations

Staircase model | Overlapping waves model

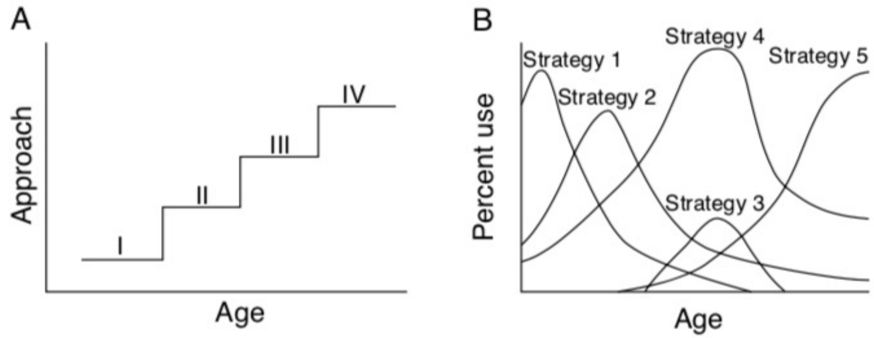


Table 1. Addition strategies used by preschoolers

Strategy	Use of strategy to solve 3+5
Count from 1	Put up 3 fingers, usually accompanied by saying'1,2,3';
	Put up 5 fingers, usually accompanied by saying '1,2,3,4,5';
	Count all fingers, saying '1,2,3,4,5,6,7,8'.
Shortcut sum	Say '1,2,3,4,5,6,7,8' perhaps simultaneously putting up fingers.
Min	Say '5,6,7,8' or '6,7,8' perhaps simultaneously putting up fingers on each count.
Count from first	Count from first addend, saying '3,4,5,6,7,8' or '4,5,6,7,8'.
Retrieval	Say an answer and explain by saying, e.g. 'I just knew it'.

Mathematical education trains ever more fine-grained magnitude representations (mental number line), specific strategies/operations, and memorization of arithmetic facts (e.g., multiplication)

Siegler, R. S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. ***Child Development Perspectives***, ***8***(3), 144–150. http://doi.org/10.1111/cdep.12077

Siegler, R. S. (1999). Strategic development. ***Trends in Cognitive Sciences***, ***3***(11), 430–435. http://doi.org/10.1016/S1364-6613(99)01372-8

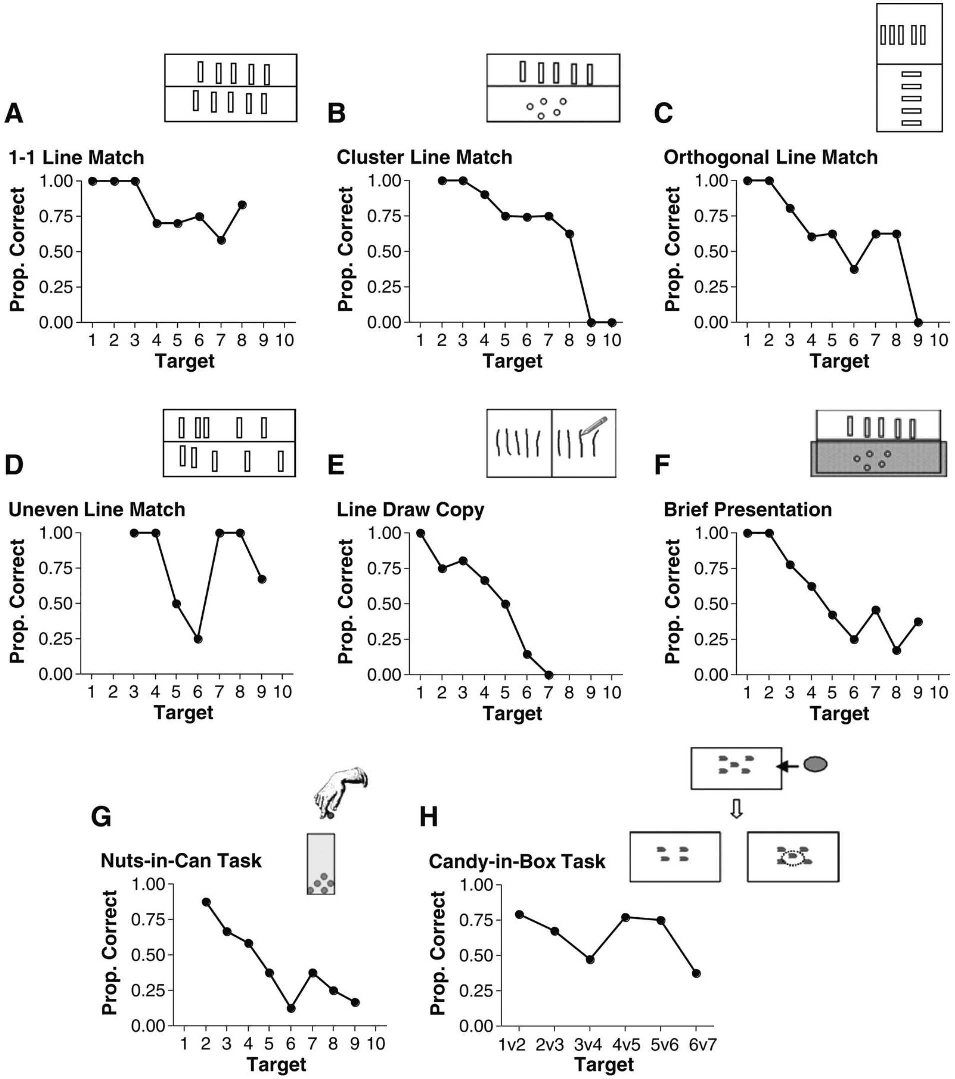
The Development of Arithmetic Abilities

Numerical Cognition Without Words: Evidence from Amazonia

Peter Gordon

Members of the Pirahã tribe use a "one-two-many" system of counting. I ask whether speakers of this innumerate language can appreciate larger numerosities without the benefit of words to encode them. This addresses the classic Whorfian question about whether language can determine thought. Results of numerical tasks with varying cognitive demands show that numerical cognition is clearly affected by the lack of a counting system in the language. Performance with quantities greater than three was remarkably poor, but showed a constant coefficient of variation, which is suggestive of an analog estimation process.

Cross-cultural studies show large differences between cultures - indicating a role of culture/language in the acquisition of numerical cognition abilities. On the one hand, evidence from «one-two-many» systems support the idea of an approximate number system (ANS) that is foundational and exists prior/independently of linguistic information for small quantities. However, large(r)-number tasks are challenging in these contexts, suggesting that culture shapes numerical abilities by providing numerical entities that can be acquired through social learning.



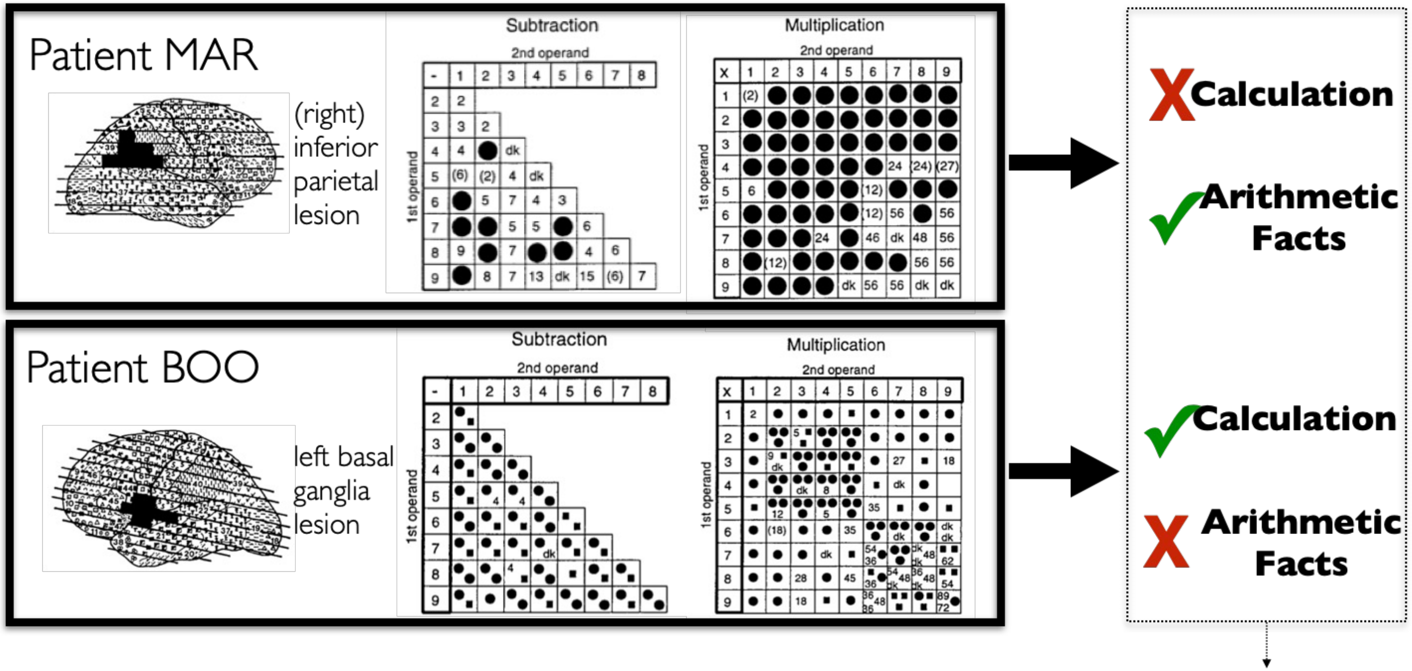
Numerical Cognition

Ontogeny	Mechanism 
Phylogeny	Adaptive Significance

The Neural Basis of Arithmetic Abilities

Subtraction is typically calculated (calculation)

Multiplication is typically retrieved from memory (arithmetic facts)



double dissociation

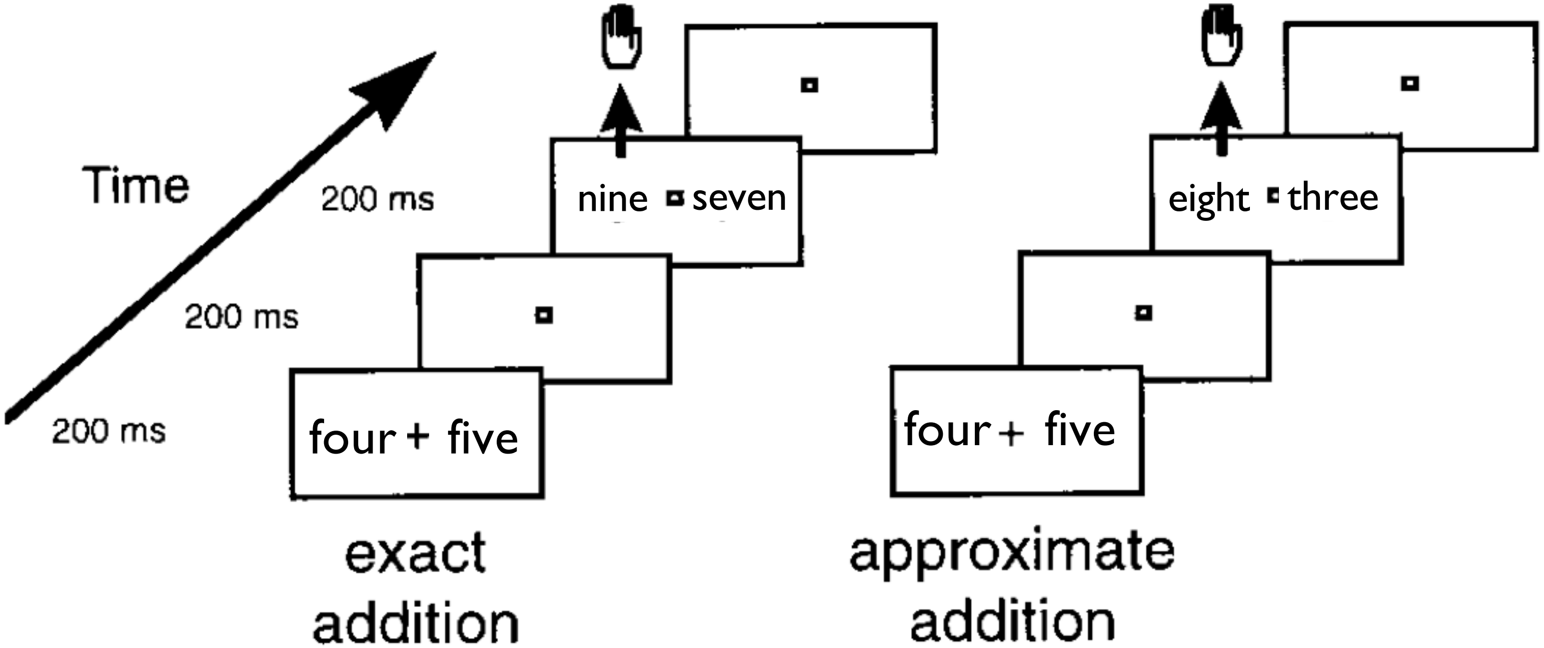
Note. Both patients were perfect in naming arabic numerals and in writing arabic numerals to dictation. Patient MAR was left-handed. Filled circles indicate correct performance. Filled squares indicate slow but correct performance.

Double Dissociation

An inferential technique by which two brain areas are functionally dissociated by two behavioral tests, each test being affected by a lesion in one zone and not the other. In a series of patients with traumatic brain injury, one might find two patients, A and B. If one can demonstrate that a lesion in brain structure X impairs function 1 but not 2 (in patient A), and further demonstrate that a lesion to brain structure Y impairs function 2 but spares function 1 (in patient B), one can make inferences about independence of function in these two brain regions.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex, 33*, 219-974.

The Neural Basis of Arithmetic Abilities



Approximate vs. Exact Arithmetic: Design of the tasks used during brain imaging. Subjects fixated continuously on a small central square. On each trial, an addition problem, then two candidate answers were flashed. Subjects selected either the correct answer (exact task) or the most plausible answer (approximate task) by depressing the corresponding hand-held button as quickly as possible. The same addition problems were used in both tasks.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. ***Science, 284***, 970-974.

The Neural Basis of Arithmetic Abilities

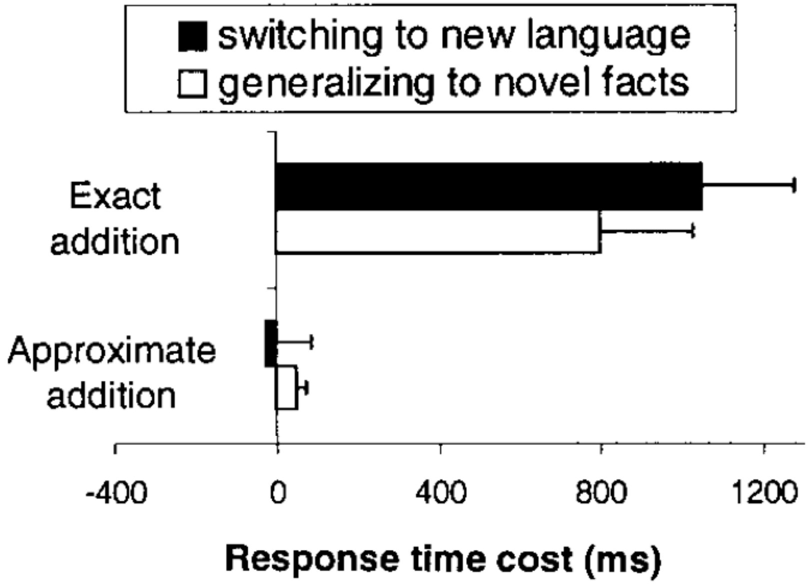
Training of Exact vs. Approximate Facts

Russian-English Bilinguals

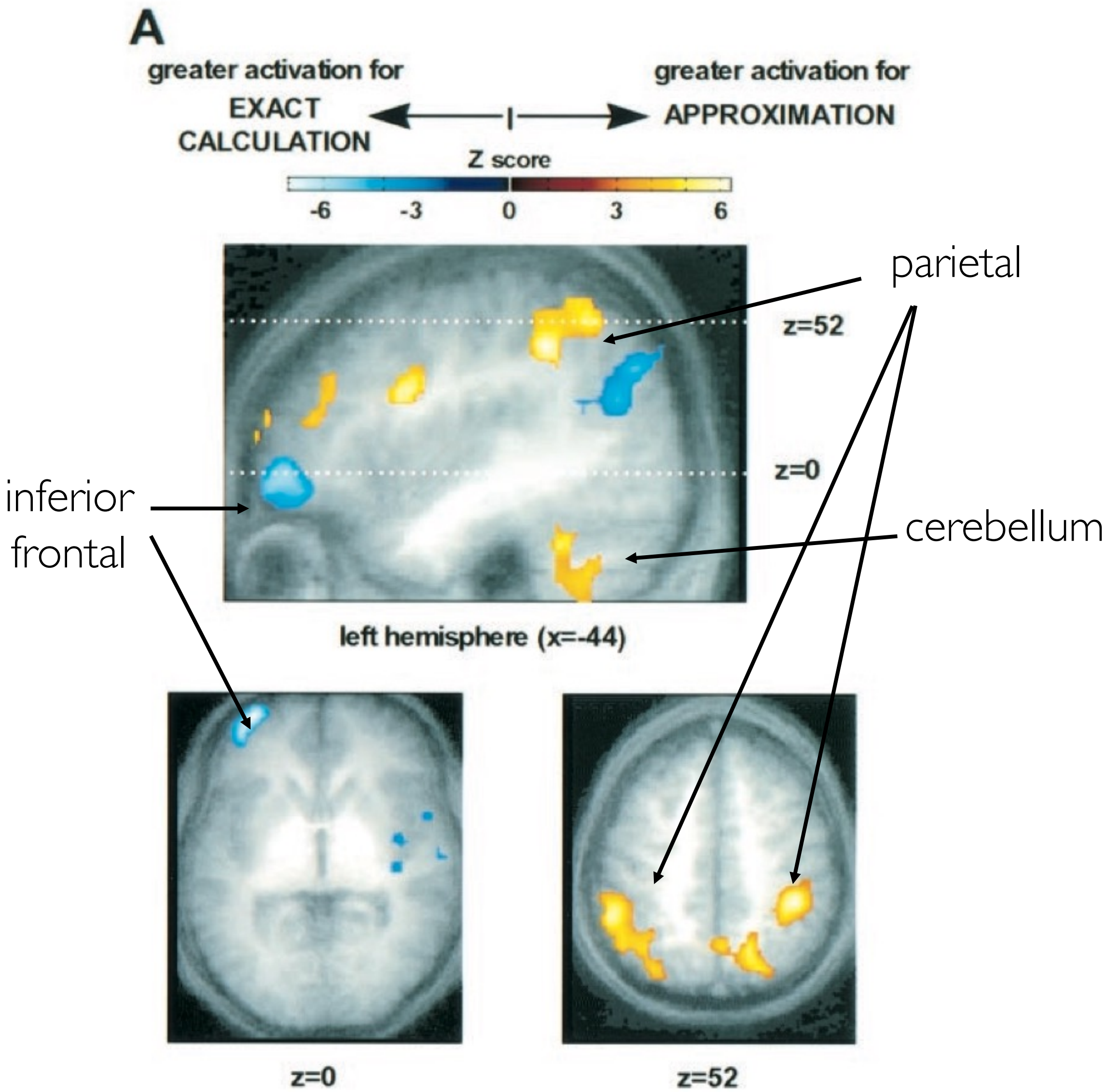
тринадцать+восемнадцать **thirteen+eighteen**



Generalization of learning new exact or approximate number facts. Mean response times (RTs) to trained problems in the trained language are subtracted from RTs to trained problems in the untrained language (**language cost**: black bars) and from untrained problems in the trained language (**generalization cost**: white bars).

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. ***Science, 284***, 970-974.

The Neural Basis of Arithmetic Abilities



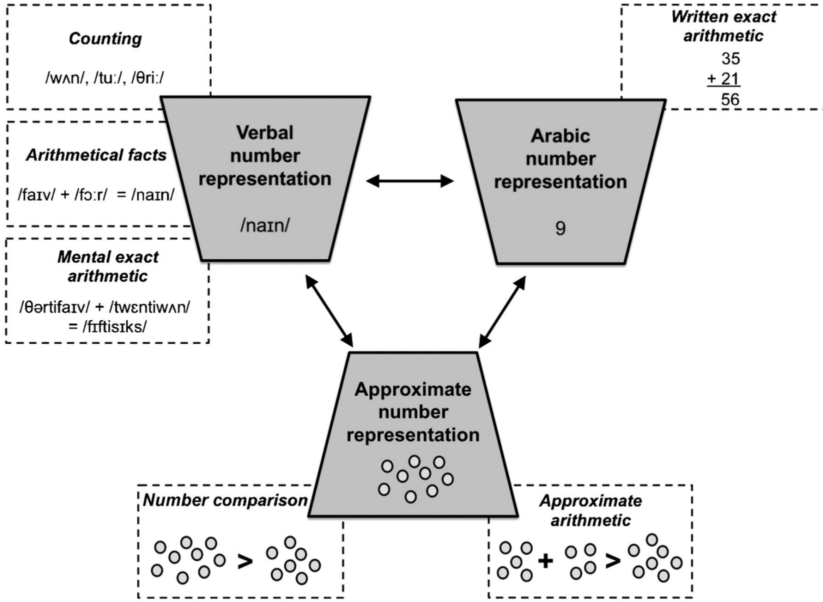
"Exact arithmetic is acquired in a language-specific format, transfers poorly to a different language or to novel facts, and recruits networks involved in word-association processes. In contrast, approximate arithmetic shows language independence, relies on a sense of numerical magnitudes, and recruits bilateral areas of the parietal lobes involved in visuo-spatial processing."

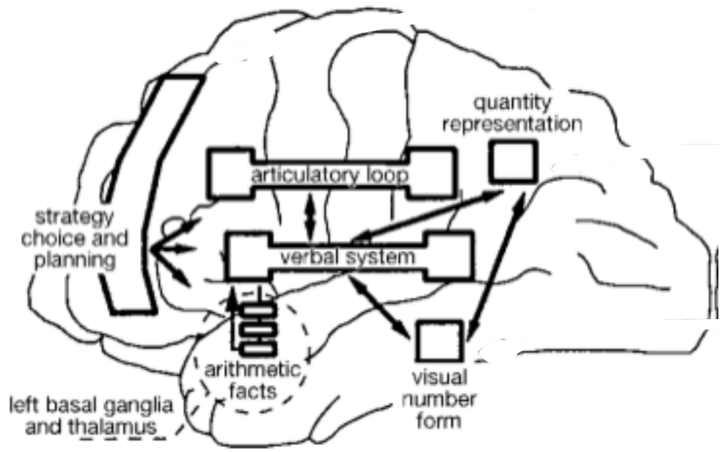
Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. ***Science, 284***, 970-974.

The Neural Basis of Arithmetic Abilities

The "triple-code model" postulates three main representations of numbers

- an analogical quantity or magnitude code, subserved by the left and right inferior parietal areas, and in which numbers are represented as distributions of activation on an oriented number line. This representation subserves semantic knowledge about numerical quantities, including proximity (e.g. 9 close to 10) and larger-smaller relations (e.g. 9 smaller than 10);
- a verbal code, subserved by left-hemispheric perisylvian areas, and in which numbers are represented as a parsed sequence of words. This representation is the primary code for accessing a rote verbal memory of arithmetic facts (e.g. "nine times nine, eighty-one").
- a visual arabic code, localized to the left and right inferior ventral occipito-temporal areas, and in which numbers are represented as identified strings of digits. This representation subserves multidigit operations and parity judgements (e.g. knowing that 12 is even because the ones digit is a 2);

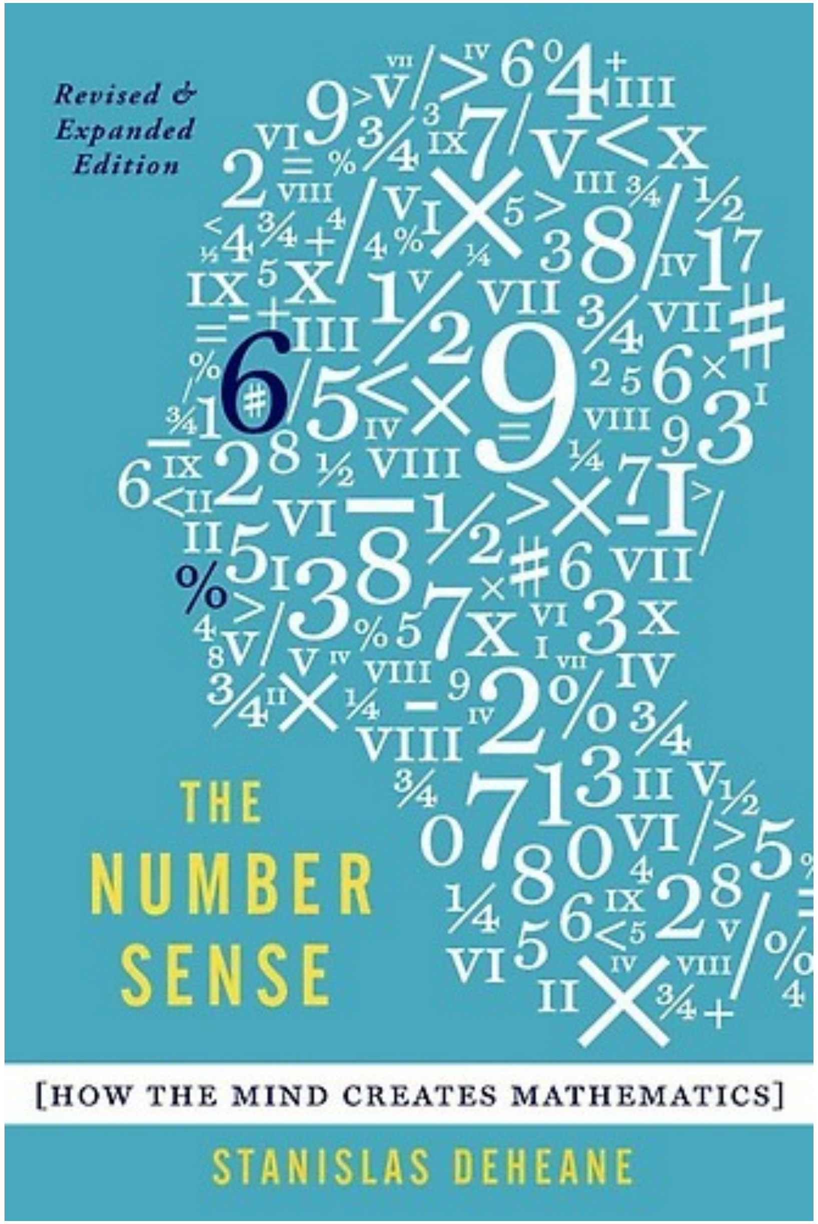




Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. ***Cortex, 33***, 219-974.

Summary

- **Adaptive Significance**: Numerical abilities are central to social and economic behavior in our species; and, today, central to engineering and technology-based occupations; current evidence suggests the gender gap in adoption of STEM professions is **not** a deep “biological” fact but is likely related to various social factors.
- **Comparative studies:** Comparative studies suggest that there is a high degree of phylogenetic continuity in the representation of (small) quantities between non-human and human primates. Similarity in the neural basis of an approximate number system - a non-linguistic system that supports the estimation of the magnitude of a group of elements.
- **Developmental approaches:** Approximate number system seems to be available universally and early in life with infants being able to represent and compare magnitudes; mathematical education goes beyond this by training ever more fine-grained magnitude representations (mental number line) and respective symbols (Arabic numerals), instruction of specific strategies/operations, and memorization of arithmetic facts (e.g., multiplication tables).
- **Neural models:** The mental representation of number, including human adults’ full arithmetic skills, include aspects linked to visual processing (e.g., representation of magnitudes through Arabic numerals), spatial cognition (e.g., sense of magnitudes), and language (e.g., arithmetic facts), with specific neural underpinnings in temporal-occipital areas, parietal cortex, and inferior frontal gyrus/basal ganglia, respectively.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THE NUMBER SENSE
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